MTTC Elementary Math Prep-Solutions
SVSU Math and Physics Resource Center

1) A student wants to find the sixth root of 64 using a calculator. However, he does not know how to perform the sixth root on his calculator. He does know how to perform the square root and the cube root. How can the student find the sixth root of 64 using only the square root and cube root operations?

a) He can take the square root of 64 six times in succession.
b) He can take the square root of 64 followed by the cube root.
c) He can take the square root of 64 three times in succession.
d) He can take the cube root of 64 twice in succession.

Answer: B) Computing the sixth root of a number is equivalent to raising the base number to the one-sixth power. We need to find the equivalent action to $64^{\frac{1}{6}}$. Below is the calculation of each answer choice.

a) If the student performs this action he will be computing. $\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{64}}}}}}$

Or $(((((64^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2}$ by multiplying the powers we reach $64^{1/64} \neq 64^{1/6}$

b) $\sqrt[3]{\sqrt{64}} = (64^{1/2})^{1/3}$ by multiplying powers $= 64^{1/6} = \sqrt[6]{64}$ Correct Answer.

c) $\sqrt{\sqrt{\sqrt{64}}} = ((64^{1/2})^{1/2})^{1/2} = 64^{1/8} \neq 64^{1/6}$

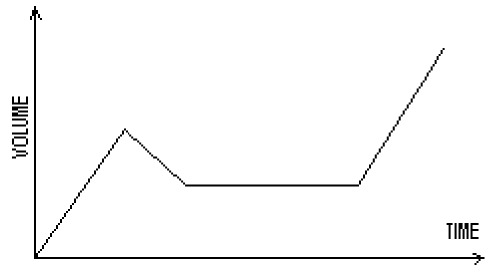
d) $\sqrt[3]{\sqrt[3]{64}} = (64^{1/3})^{1/3} = 64^{1/9} \neq 64^{1/6}$

2) Below is a sequence of numbers;
2, 3, 5, 8, 13, 21, …
If this pattern continues, how many prime numbers will there be in the first 10 terms of the sequence?

a) 3
b) 2
c) 5
d) 4

Answer: C) If the pattern continues until there are 10 terms, the sequence will be,
2,3,5,8,13,21,34,55,89,144 (Fibonacci Sequence). Add the two previous numbers to make the next number in the sequence. The highlighted and underlined terms are prime numbers. (Primes are divisible by only 1 and itself) There are 5 prime numbers in the first 10 terms. (C) is the correct answer.

3) A small swimming pool is being filled using a garden hose. The following graph depicts the volume of water in the pool with respect to time.
Which of the following statements best describe the effort to fill the pool?



a) Filling increased steadily for a time, decreased, went at a steady rate, and then filled more quickly than at the start.

b) Filling increased at a steady rate for a time, stopped, increased at a steady rate, and then stopped again.

c) Filling went at a steady rate for a time, the pool began to lose water, filling stopped for a time, and then filled again at a steady rate.

d) Filling began steadily, slowed down, went at a steady rate, and then stopped completely.

Answer: C) The graph begins as a steady rise in volume over time, which signals the pool is being filled at a steady flow rate. The downward motion of the graph indicates that the volume is decreasing in the pool, or water is being drained from the pool. The flat area signifies no change in the volume over time, or water is neither entering nor exiting the pool. The final upward motion tells us that the volume of the pool is increasing, or the pool is filling up with water, but at the same rate as the beginning. By this analysis, the answer is (c).

4) The density of a right circular cone with a height of 8 cm and diameter of 5 cm is 2.6 g/cm^3. What is its mass?

a) $\frac{160}{3}\pi$ grams

b) 60 π grams

c) $\frac{130}{3}\pi$ grams

d) $\frac{100}{3}\pi$ grams

Answer: C) Since $Density = \frac{Mass}{Volume}$, we have $D \cdot V = M$. The density is given, thus we only need to calculate the Volume. The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is half of the diameter. $V = \frac{1}{3}\pi(2.5)^2(8)$, V= $\frac{50\pi}{3}$ Therefore, $(2.6)\left(\frac{50\pi}{3}\right) = \frac{130\pi}{3}$, so the answer is (c).

5) The graph of the function, $f(x) = 3x^9 - 4x^2 + 6$ has

a) Origin symmetry

b) X-axis symmetry

c) Y-axis symmetry

d) No symmetry

Answer: D) Recall that symmetry occurs when a figure can be flipped, folded, or rotated and all points perfectly overlap. This function does not have y-axis symmetry since the end behavior of the function goes in opposite directions due to the fact that it is odd, and because $f(-x) \neq -f(x)$. It does not have x-axis symmetry since it is a polynomial and therefore a function of x. In other words it passes the vertical line test, so there are no pairs of points with the same x-coordinate, and therefore cannot have x-axis symmetry. It does not have symmetry about the origin since for every pair (x, y) there is not an equivalent $(-x, -y)$ pair (the pair of $(1,5)$ and $(-1,-7)$ is the counter example). Thus, the function has no symmetry, so the answer is (d).

6) A walker and a biker leave from the same spot at the same time. The walker travels due east at a steady rate of 2 mph and the biker travels north at a steady rate of 7 mph. Which equation must be solved to determine the distance between the two after 2 hours?

a) $4^2 + 14^2 = x^2$

b) $d = \tan^{-1}\left(\frac{7}{2}\right)$

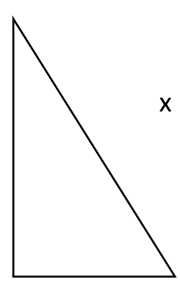
c) $a = \sqrt{4^2 + 14^2 - 2(4)(14)\cos 135°}$

d) $d = \cos^{-1}\left(\frac{2}{7}\right)$

Answer: A) Create a triangle representing the walkers and bikers paths. The walker is moving east at 2 mph for 2 hours or a distance of 4 miles. The bike is moving north at 7mph for 2 hours or a distance of 14 miles. Their positions after two hours are depicted by the triangle.

7mph*2 hours

= 14 miles



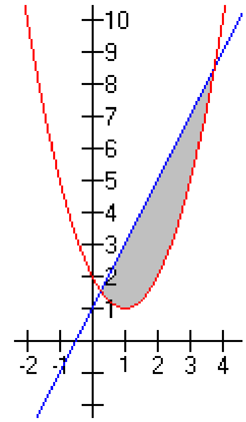
x miles

2mph*2 hours

= 4 miles

Since the triangle is right, Pythagorean's Theorem is used, $a^2 + b^2 = c^2$. So the solution is $4^2 + 14^2 = x^2$, which is (a).

7) Given the graphs: $f(x) = 2x + 1$ and $g(x) = x^2 - 2x + 2$, which of the polynomial functions models the vertical distance between the two functions, specified by the shaded region in the figure?



a) $h(x) = -x^2 + 4x - 1$
b) $h(x) = -x^2 + 3$
c) $h(x) = x^2 - 4x + 1$
d) $h(x) = -x^2 - 1$

Answer: A) The vertical distance between the two functions is found by subtracting the functions. Since the linear function, $f(x)$, is greater than the quadratic function, $g(x)$, in this interval, $f(x) - g(x) = (2x + 1) - (x^2 - 2x + 2) = 2x + 1 - x^2 + 2x - 2 = -x^2 + 4x - 1$, which is (a).

8) A frog attempts to leap across a small pond. On his first jump, he jumps $\frac{5}{8}$ of the total distance. Every jump thereafter, he only manages to jump $\frac{5}{8}$ of the remaining distance. Which of the following sums describes the distance that the frog has jumped?

a) $1 - \left[\left(\frac{5}{8}\right) + \left(\frac{5}{8}\right)^2 + \left(\frac{5}{8}\right)^3 + \cdots\right]$
b) $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \frac{5}{64} + \cdots$
c) $\frac{5}{8}\left[1 + \frac{3}{8} + \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^3 + \cdots\right]$
d) $\frac{5}{8} + \left(\frac{5}{8}\right)^2 + \left(\frac{5}{8}\right)^3 + \left(\frac{5}{8}\right)^4 + \cdots$

Answer: C) For each iteration the frog jumps $\frac{5}{8}th$ of the remaining distance. If we let the total distance be 1 unit, then the remaining distance prior to the first jump is 1 unit. After the first jump, prior to the second jump, the remaining distance is $\frac{3}{8}$ units. On the second jump the frog jumps $\frac{5}{8}th$ of this $\frac{3}{8}$ units, so the remaining distance is $\frac{3}{8}$ of the original $\frac{3}{8}$ units or $\left(\frac{3}{8}\right)^2$. So, the remaining distance after nth jump is $\left(\frac{3}{8}\right)^n$. So, the total distance jumped is $\frac{5}{8}$ times the sum of the remaining distances after each jump. The answer is c, because the term in brackets is the sum of the remaining distances after each jump.

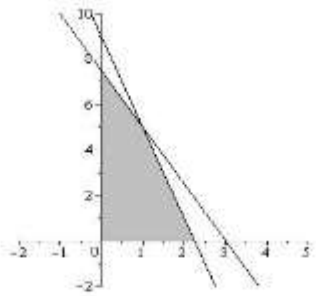
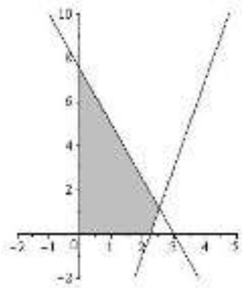
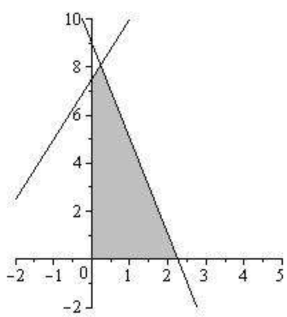
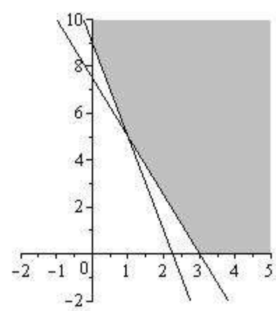
9) For the sets; $A = \{x | x \text{ is a rational number}\}$, $B = \{x | x \text{ is an irrational number}\}$, and $C = \{x | x \text{ is a real number}\}$. Which of the following is true?

a) $A \cap B = C$
b) $A \cup C = B$
c) $A \cap C = B$
d) $A \cup B = C$

Answer: D) The set of real numbers consists of the rational numbers and the irrational numbers. Since A is the set of rational numbers and B is the set of irrational numbers, their union represents the set of real numbers, the set notation is represented by (d).

10) Which figure is the graph of all solutions to the following system of inequalities, given that

$x \geq 0$ and y ≥ 0 ; $\quad \begin{cases} 28x + 7y \leq 63 \\ 5x \leq 15 - 2y \\ -2y \geq 8x - 18 \end{cases}$

a) 	c) 
b) 	d) 

Answer: A) Simplifying the inequalities by solving for y, we have the new system $\begin{cases} y \leq -4x + 9 \\ y \leq \frac{-5}{2}x + \frac{15}{2} \\ y \leq -4x + 9 \end{cases}$

The first and third inequalities represent the same inequality ($y \leq -4x + 9$). So there are only two boundaries besides the x-axis and y-axis. Since these two boundaries both have negative slopes, answers (b) and (c) are eliminated. The two remaining answers are possibilities, because the inequality $y \leq -4x + 9$ has intercepts of $(0, 9)$ and $(\frac{9}{4}, 0)$, and the inequality $y = \frac{-5}{2}x + \frac{15}{2}$ has intercepts of $(0, \frac{15}{2})$ and $(3, 0)$. Since both graphs are identical with the exception of which half-plane is shaded, use the test point $(0, 0)$ to decide. Since (0,0) makes both inequalities true, the origin is a solution and the shading must be beneath the lines. So, the answer is (a).

11) Let $A = \{1, 4, 6, 7, 9, 10\}$ and $B = \{2, 4, 5, 8, 10\}$. What is the set $A \cap B$?

a) $\{1, 6, 7, 9\}$

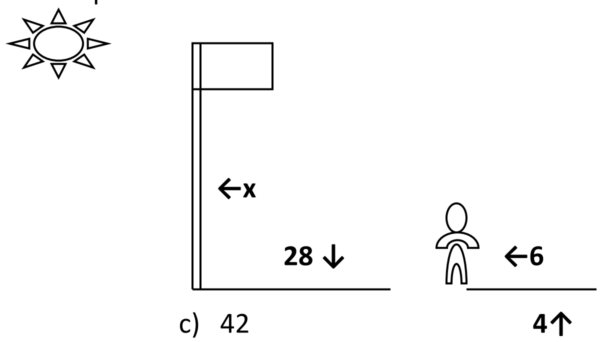
b) $\{4, 10\}$

c) $\{2, 5, 8\}$

d) $\{1, 2, 6, 7, 8, 9\}$

Answer: B) The symbol "∩" stands for the intersection of two or more sets. It is the set of numbers or elements present in both sets. For A and B, the numbers or elements that appear in both sets are 4 and 10, thus the answer is (b).

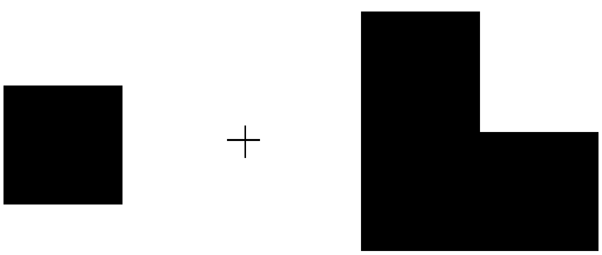
12) If the 6 foot man casts a shadow 4 feet long, and at the same time of day the flag pole casts a shadow 28 feet long, how tall is the pole?



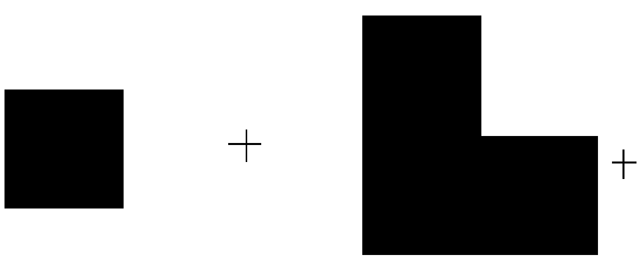
a) 24
b) 18.67
c) 42
d) 0.86

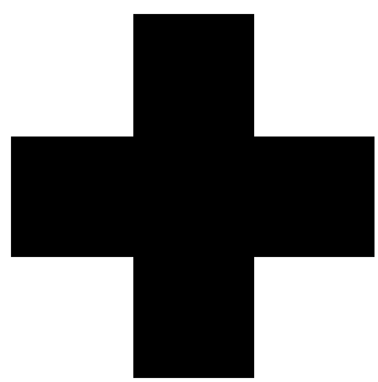
Answer: C) Similar right triangles and proportions can be used to solve this problem. A right angle is formed with the flagpole and its shadow as well as with the man and his shadow. So the solution can be found by equating ratios of the lengths of corresponding sides to form the following proportion. $\frac{\text{Height of man}}{h\text{eight of flag pole}} = \frac{\text{length of man}'\text{ s shaddow}}{\text{length of flagpoles}'\text{ s shaddow}} \to \frac{6}{x} = \frac{4}{28} \to$
$6 * 28 = 4 * x \to x = \frac{6*28}{4} \to x = 42$. The answer is (c).

13) An elementary school math class is using square tiles to learn about fractions. The problem below was created for the students to solve.



$= \frac{1}{2}$





= ?

What is the solution to this problem?

a) $1\frac{1}{2}$
b) $\frac{3}{4}$
c) 1
d) $\frac{9}{8}$

Answer: D) Since the first equation equals one-half, each of the four square must equal one-eighth. The second equation adds five-eighths to $\frac{1}{2}$. The numerical equation is represented by $\frac{1}{2}+\frac{5}{8}=\frac{4}{8}+\frac{5}{8}=\frac{9}{8}$. The answer is (d).

14) There exists a quadratic function $g(x)$ that has zeros at $x = 2$ and $x = -3$. The function $g(x)$ also has a y-intercept at -6. The function $g(x)$ is then translated long the x-axis by +2 to create $f(x)$. Determine the equation of $f(x)$, the function after translation.

a) $f(x) = x^2 + x - 4$
b) $f(x) = x^2 + x - 8$
c) $f(x) = x^2 - 3x - 4$
d) $f(x) = 2x^2 + 2x - 12$

Answer: C) The original quadratic function g(x) has zeros at $x = 2$ and $x = -3$. That means $x - 2 = 0$ and $x + 3 = 0$. So, $g(x) = a(x-2)(x+3) = a(x^2 + x - 6)$. Since $g(0) = a(-6)$ and the y-intercept of $g(x) = -6$, then $a = 1$. Translation along the x-axis by $+2$ means shifting horizontally by two units to the right. So the zeros of the original quadratic, $g(x)$, shift two places to the right. Thus the zeros for $f(x)$ are $x = 2 + 2 = 4$ and $x = -3 + 2 = -1$. Thus the new quadratic function is $f(x) = a(x+1)(x-4)$. Since $a = 1, f(x) = 1(x+1)(x-4) = x^2 - 3x - 4$. The answer is (c).

15) Consider a ceiling fan that has 5 blades, as shown in the figure below. What is the angle measurement, in degrees, between the axes's of center of two consecutive fan blades?

a) $\frac{\pi}{2}$
b) $\frac{2}{5}\pi$
c) 72°
d) 36°

Answer: C) First, since the answer must be in degrees, we can eliminate (a) and (b) as the answer. Now, the ceiling fan forms a regular pentagon. The five central angles of a regular pentagon have the same measure, found by dividing the total number of degrees (360) by 5. The central angle, and the angle between the blades, is $\frac{360}{5} = 72°$, which is (c).

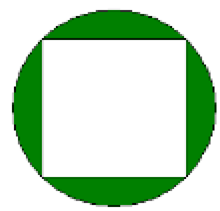
16) A second degree polynomial, $P(x)$ has zeros at $x = 7$ and $x = 9$. Which function represents $N(x) = P(x+2)$?

a) $N(x) = x^2 - 16x + 35$
b) $N(x) = x^2 - 20x + 99$
c) $N(x) = x^2 - 12x + 35$
d) $N(x) = x^2 - 12x + 99$

Answer: C) The zeros of P(x) are $x = 7$ and $x = 9$. Thus, $P(x) = a(x-7)(x-9) = a(x^2 - 16x + 63)$. Therefore, $N(x) = P(x+2) = (x+2)^2 - 16(x+2) + 63 = x^2 + 4x + 4 - 16x - 32 + 63 = x^2 - 12x + 35$, which is (c).

17) A landscaper is designing a circular garden that encloses a square fountain as shown below. If one bag of fertilizer treats 20 square feet, how many bags must he use to treat the entire garden if the fountain is 20 feet wide?

a) 1 bag
b) 12 bags
c) 21 bags
d) 9 bags



Answer: B) In order to find the area of the shaded region (the garden), we need to find the area of the square and subtract it from the area of the circle. Recall that $Area_{circle} = \pi r^2$ and that $Area_{square} = s^2$. Since the diameter of the circle is the diagonal of the square, Pythagorean's theorem can be used to find the diameter. Since the side length of the square are 20 feet, $20^2 + 20^2 = c^2$, where c is the length of the diameter. Thus, the radius is one-half of the diameter, which is $10\sqrt{2}$. Now the area of the shaded region can be calculated.

$$Area_{circle} - Area_{square} = \pi r^2 - s^2 = \pi\left(10\sqrt{2}\right)^2 - (20)^2 = 200\pi - 400 \approx 228.318$$

The Area of the garden is about 228 square feet. Each bag covers 20 square feet. The number of bags needed is $\frac{228}{20} \approx 11.4$. Therefore, the landscaper needs 12 bags for the garden, which is (b).

18) Consider the functions $f(x) = \sqrt{x+2}$ and $g(x) = x + 12$. What is the domain of $(f \circ g)(x)$?

a) $x \geq 0$
b) $x \geq 14$
c) $x \geq -14$
d) $x \leq 14$

Answer: C) Recall in the composition $(f \circ g)(x)$ that $g(x)$ becomes the input for $f(x)$. Then $(f \circ g)(x) = f(x+12) = \sqrt{(x+12)+2} = \sqrt{x+14}$. The domain of the compositions is the intersection of the domains of the input function and the final expression. The domain of $g(x)$ is all real numbers. Since a square root is defined only for radicands greater than or equal to zero, the domain of $\sqrt{x+14}$ must satisfy the inequality $x + 14 \geq 0$. Thus the domain of $(f \circ g)(x)$ is ≥ -14, which is (c).

19) Suppose the following statement is true: If it is raining outside, Jethro carries his umbrella. Which of the following statements is definitely true?

a) If it is not raining outside, Jethro does not carry his umbrella.

b) If Jethro carries his umbrella, it is raining outside.

c) If Jethro does not carry his umbrella, then it is not raining outside.

d) If it is not raining outside, Jethro carries his umbrella.

Answer: C) Recall that for any given true conditional statement, the contrapositive is also true. The contrapositive of "If A, then B" is "If not B, then not A", which is (c).

20) A club has 12 female members and 14 male members. How many different ways can a committee of 6 be formed if an equal number of men and women must be selected?

a) 584

b) 80,080

c) 2,882,880

d) 3,504

Answer: B) This is a combination problem, since selection order doesn't matter in forming a committee. Since we need a group of 6 members with equal number of men and women, there must be three of each. There are 12 women and 14 men to choose from. Therefore, the total possible ways the committee can be chosen is ${}_{12}C_3 \cdot {}_{14}C_3 = \frac{12!}{3!(12-3)!} \cdot \frac{14!}{3!(14-3)!} = 220 \cdot 364 = 80{,}080$ which is (b).

21) Solve the following system of equations.

$$\begin{cases} 3x + 2y + z = 2 \\ 2x - y - z = 1 \\ x + y = 1 \end{cases}$$

a) $x = 2, y = 1, z = 1$

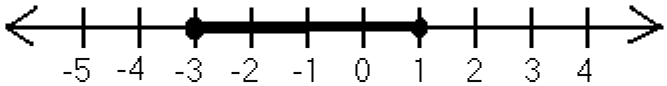
b) $x = 1, y = 1, z = -3$

c) $x = \frac{1}{2}, y = 1, z = -\frac{1}{2}$

d) $x = \frac{1}{2}, y = \frac{1}{2}, z = -\frac{1}{2}$

Answer: D) First use substitution. Since $x + y = 1$, $x = 1 - y$. By substituting this expression for x in the first two equations, the system becomes $\begin{cases} 3 - 3y + 2y + z = 2 \\ 2 - 2y - y - z = 1 \end{cases} \rightarrow \begin{cases} 3 - y + z = 2 \\ 2 - 3y - z = 1 \end{cases}$. Then eliminate z by adding the two remaining equations together to obtain the equation $5 - 4y = 3$. Upon solving, $= \frac{1}{2}$. Next substitute $y = \frac{1}{2}$ into $3 - y + z = 2$ and solve to obtain $z = -\frac{1}{2}$. To solve for the remaining variable, x, substitute $y = \frac{1}{2}$ into the original 3rd equation, $x + y = 1$, and solve to obtain $x = \frac{1}{2}$. So, the answer is (d).

22) Consider the solution set shown on the number line below.



Which of the following absolute value inequalities has the same solution set as the one shown?

a) $|x-1| \leq 2$

b) $|x+1| \geq 2$

c) $|x+1| \leq 2$

d) $|x-1| \geq 2$

Answer: C) This is a compound inequality expressed as $-3 \leq x \leq 1$. The midpoint is halfway between, which is $\frac{-3+1}{2} = \frac{-2}{2} = -1$. From the center at -1, the points at -3 and 1 are two units away. Using an absolute value inequality to represent the solution set drawn, $|x-(-1)| \leq 2 = |x+1| \leq 2$, which is (c).

23) Jack goes to Lucky's Casino every Friday after he gets paid. He makes the same bet every week and has decided that he wins 30% of the time. Every week, he either loses \$10 or wins a net of \$25 (including his bet of \$10). Using this information, how much can Jack expect to net, on average, each week?

a) \$0

b) \$0.50

c) \$7.50

d) \$2.50

Answer: B) To calculate the net expected value, compute the expected gain and subtract the expected loss. The probability of winning is 0.3 while the probability of losing is 0.7 (recall that the total probability of all outcomes is 1). Therefore, $expected\ net\ value = expected\ gain - expected\ loss = 0.3 * 25 - 0.7 * 10 = 0.5$, which is (b).

24) Consider the sets S and R below.

$S = \{x|\ x \text{ is an integer}\}$

$R = \left\{x|\ x \text{ is in the form of } \frac{a}{b} \text{ where } a \text{ and } b \text{ are integers and } b \neq 0\right\}$

Which of the following statements about S and R is true?

a) Elements of $S \cap R$ are integers

b) Elements of $S \cup R$ are complex numbers

c) Elements of $S \cap R$ are complex numbers

d) Elements of $S \cup R$ are irrational numbers

Answer: A) The sets S and R describe the set of integers and the set of rational numbers respectively. Both are subsets of the real numbers, which eliminates (b) and (c). All integers are rational, which eliminates (d). So, the answer is (a), the intersection of the two sets is an integer.

25) Which of the following is <u>not</u> a possible rational root of the polynomial
$100x^6 + 21x^5 - 40x^3 - 3x^2 + 17x - 100$

a) $\frac{5}{2}$

b) $\frac{3}{4}$

c) 20

d) $\frac{4}{5}$

Answer: B) To find the possible rational zeros, use the rational zeros theorem, which states that potential rational zeros are the ratios of the factors of the constant term over the factors of the leading coefficient. The factors of the leading coefficient 100 are $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm25, \pm50, \pm100$. The factors of constant term -100 are the same. The only answer choice that does not have a factor of –100 is (b), which is $\frac{3}{4}$.

26) A student is using a computer program to graph the equation of a line in the form of $y = mx + b$. If both m and b are elements of the set $\{1,3,5\}$, how many distinct lines can be drawn?

a) 3

b) 6

c) 8

d) 9

Answer: D) The numbers $\{1,3,5\}$ represent the possible values for m and b, where the value can be repeated. The combinations for $\{m, b\}$ are $\{1,1\}, \{1,3\}$ $\{1,5\}, \{3,1\}, \{3,3\}, \{3,5\}, \{5,1\}, \{5,3\}$, and $\{5,5\}$, totaling $3 \cdot 3 = 9$ unique lines. Thus (d) is the answer.

27) A pig farmer is interested in whether or not his pigs approve of the new slop he obtained from the market today. He takes a random sample from four different pens. The sample size and the population of each pen are given in a table below.

Pen	Population	Sample
1	92,000	500
2	60,000	750
3	35,000	600
4	25,000	400

Which pen has the highest probability of correctly estimating the percentage of pigs that approve the slop?

a) Pen 1
b) Pen 2
c) Pen 3
d) Pen 4

Answer: C) The highest ratio between the sample and the population gives the highest probability at correctly estimating the percentage of pigs. By dividing each sample by the population we get, $Pen\ 1 = \frac{500}{92{,}000} \approx 0.005435;\ Pen\ 2 = \frac{750}{60{,}000} \approx 0.0125;\ Pen\ 3 = \frac{600}{35{,}000} \approx 0.017143;$
$Pen\ 4 = \frac{400}{25{,}000} \approx 0.016$. The highest ratio is Pen 3, which is (c).

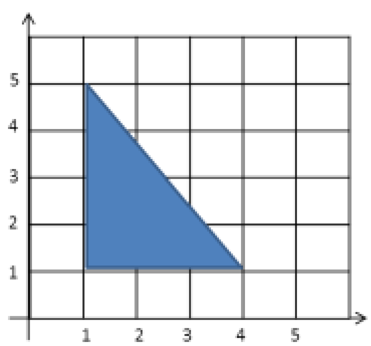
28) The (x, y), coordinates of the vertices of a polygon are given in the table below.

Vertices
(1, 1)
(1, 5)
(4, 1)

a)) 8
b) 10
c) 6
d) 4

Find the area of the shape.

Answer: C) First graph the vertices and sketch the shape.



The polygon is a right triangle. Use the formula $A = \frac{1}{2}bh$ to find out the surface area. The base measures 3 and the height measures 4. Therefore: $\frac{1}{2}(3)(4) = 6$, which is (c).

29) Bartholomew works every fifth day and Gretchen works every fourth day. How frequently do they work together?

a) Every 10th day
b) Every 15th day
c) Every 20th day
d) Every 25th day

Answer: C) The lowest common multiple between four and five is 20, so every multiple of 20 days is when they work together.

30) On a calculator, a student starts with 5 and takes the cube root three times in succession. This is not equivalent to which of the following.

a) $5^{\frac{1}{27}}$

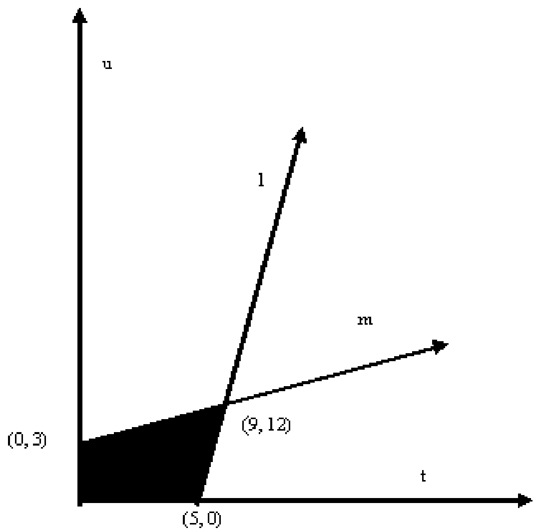
c) $5^{0.\overline{037}}$

b) $\sqrt[27]{5}$

d) $\left(5^{\frac{1}{3}}\right)^{3}$

Answer: D) Taking the cube root of the same number can be represented as $\left(\left(5^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}$ which is equivalent to $5^{\frac{1}{27}}$. This excludes (a) and (b) as answers. Now, $\frac{1}{27} = 0.\overline{037}$, which excludes (c). So the correct answer is (d). (Note that $(5^{\frac{1}{3}})^{\,3} = 5$)

31) Use the diagram below to answer the question that follows.



If $u \geq 0$ and $t \geq 0$, which of the following system of equations corresponds to the shaded portion of the graph?

a) $\begin{cases} u - 3t \geq -15 \\ 2u - 2t \leq 6 \end{cases}$

b) $\begin{cases} u + 3t \geq 10 \\ 2u - t \leq 6 \end{cases}$

c) $\begin{cases} 2u - 3t \geq 3 \\ u - t \leq 5 \end{cases}$

d) $\begin{cases} u - 3t > -15 \\ u - t < 6 \end{cases}$

Answer: A) In this problem, $t = x$ and $u = y$ on the coordinate plane. Using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, we can find the slope of line m, which is $\frac{12-3}{9} = 1$. Since $(0{,}3)$ is the intercept, the line m is represented by the equation $u = 1t + 3$, which can also be written as $2u - 2t = 6$. Since the test point $(0{,}0)$ is in the shaded region, the inequality is $2u - 2t \leq 6$. The slope of line l is $\frac{12}{9-5} = 3$. Using this slope, the point $(5{,}0)$, and the point-slope equation, line l can be written as $u - 0 = 3(t - 5)$, which can be rewritten as $u - 3t = -15$. Since the test point $(0{,}0)$ is in the shaded region, the inequality is $u - 3t \geq -15$. So, the system is (a).

32) Which polygon cannot be tessellated?

a) Equilateral Triangle

b) Regular Pentagon

c) Square

d) Regular Hexagon

Answer: B) A figure can only be tessellated if the interior angle measure can divide 360°. The interior angle of a regular polygon equals $\frac{(n-2)180°}{n}$. The interior angles of an equilateral triangle are 60°, of a regular pentagon are 108°, of a square are 90°, and of a hexagon are 120°. Only 108°, the measure of the interior angle of a pentagon, does not divide 360° evenly.

33) A hand is two feet from a flashlight and two yards from a screen forming a hand puppet shadow that is twenty inches tall. How tall is the hand that is forming the shadow?

a) 10 inches

b) $6.\overline{6}$ inches

c) $\frac{20}{3}$ inches

d) 5 inches

Answer: D) The triangles created by the hand and the flashlight and the shadow and flashlight are similar, thus their corresponding sides are proportional. Therefore, a proportion can be used to find the size of the hand. $\frac{\text{2 feet}}{x} = \frac{\text{2 yards and 2 feet}}{\text{20 inches}}$. However, the units are inconsistent, so conversions must be used to solve the proportion. Since $1 \text{ yard} = 36 \text{ inches}$, and $1 \text{ foot } = 12 \text{ inches}$, $\frac{2\cdot 12\ inches}{x} = \frac{2\cdot 36+2\cdot 12\ inches}{20\ inches} \leftrightarrow \frac{24\ inches}{x} = \frac{96}{20} \leftrightarrow 480\ inches = 96x \leftrightarrow \frac{480}{96} = x \leftrightarrow x = 5$. The answer is (d).

34) What is the least common multiple and greatest common factor of 4200 and 6160?

a) 92400; 280

b) 9240; 280

c) 92400; 2800

d) 9240; 2800

Answer: A) The most efficient method is prime factorization. The prime factorization for 4200 is $2^3 * 3 * 5^2 * 7$. The prime factorization for 6160 is $2^4 * 5 * 7 * 11$. The LCM uses the largest exponent of each distinct factor from each list of factors. Therefore, the LCM of 4200 and 6160 is $2^4 * 3 * 5^2 * 7 * 11 = 92400$. The GCF uses the smallest exponent of each similar factor from both lists. Therefore, the GCF of 4200 and 6160 is $2^3 * 5 * 7 = 280$. The answer is (a).

35) If a triangle has side lengths of a, b and c, which relationship has to exist between the three sides for every triangle?

a) $a + b > c$

b) $ab > bc$

c) $a^2 + b^2 = c^2$

d) (a) and (c) are both correct

Answer: A) The Triangle Inequality Theorem states that the length of any one side of a triangle must be smaller than the sum of the lengths of the other two sides, regardless of the type of triangle. This relationship is described in (a). (Note: (c), which is the Pythagorean Theorem, is only valid in right triangles.)

36) What is the best type of chart to use to compare two different sets of geometry test scores?

a) Stem and leaf plot

b) Box plot

c) Scatter plot

d) Pie charts

Answer: B) The Box plot allows quick comparison of the measure of center of the distributions (medians) and the variation in each distribution. Both the center and variation in the distributions must be considered when comparing distributions. Stem and leaf plot would also show the variation, but the measure of center would be more difficult to visualize.

37) Let $f(a) = \frac{2}{3}(3a + 9) - 3$. Now, $g(b)$ is parallel to $f(a)$, with a y-intercept of 1. $h(c)$ is a line passing through the points (3, 5) and (5, 1). Determine where $g(b)$ intersects $h(c)$.

a) $\left(\frac{5}{2}, 6\right)$

b) $\left(\frac{5}{2}, 10\right)$

c) $(6, 10)$

d) $(6, 6)$

Answer: A) Since $g(b)$ is parallel to $f(a)$, the slopes are equivalent. Therefore, $g(b) = 2b + 1$. (Recall that the slope of $f(a) = \frac{2}{3} * 3 = 2$). Now, $h(c) = -2c + 11$. (Use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find slope and use $y - y_1 = m(x - x_1)$ to find the equation of the line). The point at which the two functions cross are when they are equal (use x for b and c). $2x + 1 = -2x + 11 \leftrightarrow 4x = 10 \leftrightarrow x = \frac{5}{2}$. Then substitute $\frac{5}{2}$ for x into $g(b)$ and get, $g(b) = 2\left(\frac{5}{2}\right) + 1 \rightarrow 6$. The point of intersection is $\left(\frac{5}{2}, 6\right)$, which is (a).

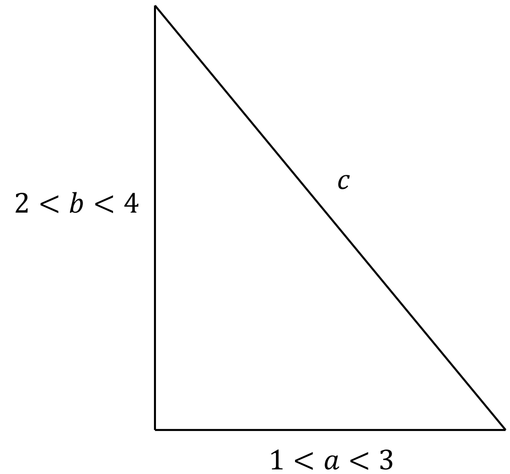
38) Given the right triangle shown below, which value for c is reasonable?

a) -5

b) 2

c) 4

d) 13



Answer: C) Since the range is given for both legs of the triangle, the range for the length of the hypotenuse can be found using Pythagorean Theorem. $1^2 + 2^2 = c^2 \rightarrow c = \sqrt{5}$ and $3^2 + 4^2 = c^2 \rightarrow c = 5$. Therefore the range of c is between $\sqrt{5}$ and 5. The only value that is between is 4, so the answer is c).

39) Simplify the following: $2[3(x+1) - (4+x)^2]$

a) $-2x^2 - 10x - 26$

b) $-2x^2 + 6x - 26$

c) $2x^2 + 22x - 26$

d) $4x^2 + 20x - 52$

Answer: A) Using proper order of operations, $2[3(x+1) - (4+x)^2] = 2[3(x+1) - (16 + 8x + x^2)]$ $= 2[3x + 3 - x^2 - 8x - 16] = 2[-x^2 - 5x - 13] = -2x^2 - 10x - 26$. If you found a different answer, check if any of the following common mistakes were made. For (b), $(4+x)^2 = 16 + x^2$, which is incorrect since exponents cannot be distributed to a sum or difference. For (c), $3(x+1) - (4+x)^2 = 3(x+1) - (16 + 8x + x^2) = 3(x+1) - 16 + 8x + x^2$, which is incorrect since the negative sign was not distributed. For (d), $2[3(x+1) - (4+x)^2] = 6(x+1) - (8x+2)^2$, which is incorrect since the two cannot be distributed before the squaring the binomial.

40) Using a base five system, what is $14_5 + 23_5$?

a) 42_5

b) 302_5

c) 37_5

d) 40_5

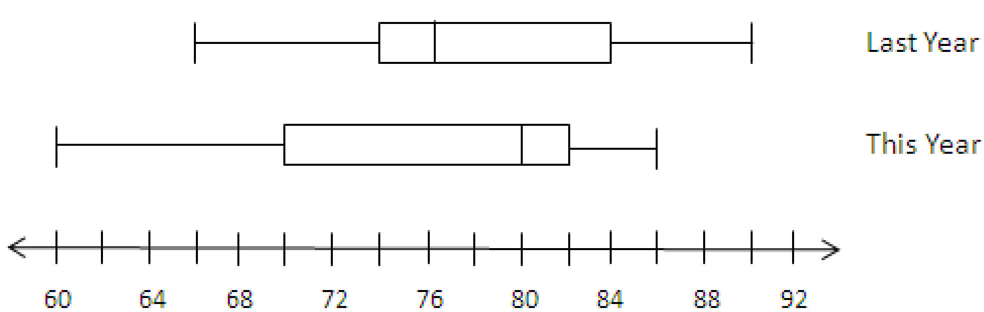
Answer: A) Since both values are in base 5, the numbers need to be added, remembering to carry when the sum of any two digits reaches five. Working with the one's place value first, upon regrouping, $4_5 + 3_5 = 5_5 + 2_5 = 10_5 + 2_5$. So, the one's place value of the sum is 2_5 and 1, representing one group of five, must be carried to the five's place value. For this reason, the sum of five's place values is $10_5 + 20_5 + 10_5 = 40_5$. Combining the one's and five's place values results in $40_5 + 2_5 = 42_5$, which is answer (a).

41) Sally's Salon sells Simply Sally Hair Spray. Sally wants to sell all of her hair spray by the end of the year, so if she has any left in stock after Thanksgiving, she puts it on sale. Sally has found that the probability of selling all of her hair spray by Thanksgiving is 70%. Sally makes $7 profit per can at full price and $4 profit per can at sale price. What profit per can, on average, can Sally expect to make?

a) $61.00
b) $6.10
c) $5.50
d) $7.00

Answer: B) The profit per can is the expected value, which equals the probability of full price times the profit of full price plus the probability of the sale price times the sale price profit, or $7(0.7) + 4(0.3) = 4.9 + 1.2 = 6.1$. Therefore, the expected profit is $6.10 per can, which is (b).

42) The plots below display math scores from a 7th grade geometry final exam this year and last year. Which of the following is an accurate statement about the results?



a) Unfortunately, the current geometry scores are not as high as last year. This year a higher percentage of students than last year had raw scores below 76.
b) Unfortunately, the current geometry scores are not as high as last year. Last year a higher percentage of students scored higher than 86.
c) Even though the median increased, because 50th percentile confidence intervals overlap, the statistical significance of the increase is in doubt without performing additional statistical tests.
d) Fortunately, the current geometry scores show statistically significant improvement because this year's median is higher than last year's median.

Answer: C) This year's median score is greater than last year's score; however, the significance of this change is in doubt, because of the wide variation in individual scores. Note that the middle 50% of each year's tests overlap, meaning the 50th percentile confidence intervals overlap. To determine if the increase in the median score is significant would require addition testing such as hypothesis test comparing the two means. So the answer is (c).

43) Use the table below to answer the question that follows.

Vertex	Coordinates
A	(1,2)
B	(5,2)
C	(5,4)

The three vertices A, B, C make up a triangle. What is the area of the triangle?

a) 5 square units

b) 4 square units

c) $2\sqrt{5}$ square units

d) 8 square units

Answer: B) To find the area of a triangle, the height and the base must be known. The distances between each of the vertices are as follows.

$$d(AB) = \sqrt{(5-1)^2 + (2-2)^2} = \sqrt{16+0} = 4$$
$$d(BC) = \sqrt{(5-5)^2 + (4-2)^2} = \sqrt{0+4} = 2$$
$$d(CA) = \sqrt{(1-5)^2 + (4-2)^2} = \sqrt{16+4} = 2\sqrt{5}$$

Using Pythagorean Theorem, it is determined that the triangle is right $(4^2 + 2^2 = 16 + 4 = 20 = \left(2\sqrt{5}\right)^2)$. Thus the legs of the triangle are the height and base. Therefore, $A = \frac{1}{2}(4 * 2) = 4\ square\ units$. The answer is (b).

44) A farmer is building a pig pen. He needs at least 10 square feet per pig. The length is 2 units longer than a constant and the width is 1 more than a constant. What are the possible values for the constant so the farmer can hold at least three pigs?

a) $x > 4$

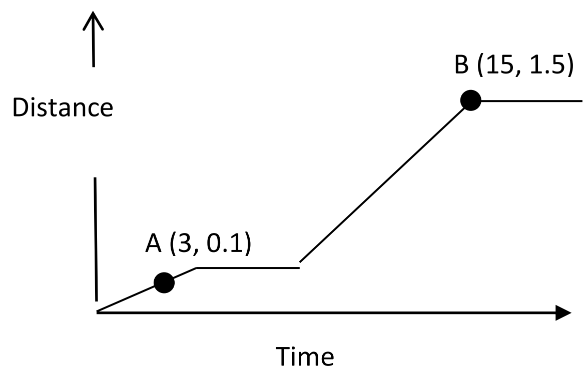
b) $x < 7$

c) $x = 4$

d) $-7 < x < 4$

Answer: A) Since the area of the pen must be larger than 10 square feet per pig, the total area must be greater than or equal to 30 square feet for three pigs. Let x be the constant. Then the dimensions of the pen are $x + 2$ and $x + 1$. The area inequality is given by $(x+2)(x+1) \geq 30$. After distributing and simplifying, the inequality becomes $x^2 + 3x - 28 \geq 0$. By factoring the quadratic, $(x+7)(x-4) \geq 0$. Solving for x, $x \geq 4$ or $x \leq -7$. Since, $x \leq -7$ produces negative lengths, this inequality is dismissed for practical reason. Thus the answer is (a).

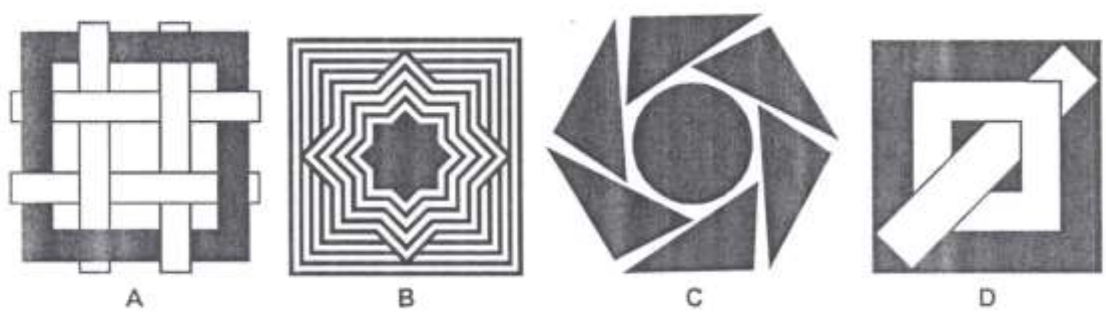
45) Bre went for a jog. Her distance as a function of time is shown in the graph. From the slope of the secant line between points A and B, what would you be able to determine about her jog?



a) How far she went between points A and B.
b) Her acceleration between points A and B.
c) Her average speed between points A and B.
d) The time it took for her to go from A to B.

Answer: C) Since the graph is of distance versus time, the slope is speed or rate of change of distance with time. Since the speed is not constant over the interval AB, the slope represents the average speed.

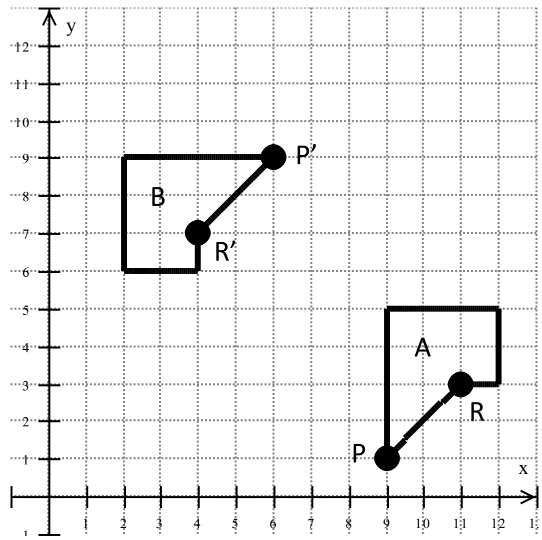
46) Which of the figures below has both reflective and rotational symmetry?



a) Figure A
b) Figure B
c) Figure C
d) Figure D

Answer: B) Figure A, C and D do not have reflective symmetry. Figure D does not have rotational symmetry. Figure B can be rotated 90°, 180°, or 270°. It has horizontal, vertical and two 45° angular lines of reflective symmetry, so the answer is (b).

47) Describe the transformation between A and B with rotation about point R.



a) Reflect over the line $y = 5$, translate left 7, rotate 90° clockwise.
b) Translate down 4, reflect over $y = 7$, rotate 90° clockwise.
c) Translate right 6, rotate 180° clockwise, reflect over $y = 5$.
d) Rotate 270° clockwise, reflect over $y = 5$, translate right 6.

Answer: A) Upon reflection over $y = 5$, the point R is moved to $(11, 7)$ and P is moved to $(9, 9)$. Upon translation left 7, point R' is moved to $(4, 7)$ and P to $(2, 9)$. Upon rotation 90° clockwise around point R, point R remains at $(4, 7)$ and P moves to $(6, 9)$.

48) A radioactive material has a half-life of 10 s. If there was 1000 mg of material at time zero, how much is left after t s?

a) $A = A_0(2)^{-t/10}$
b) $A = A_0(e)^{-t/10}$
c) $A = log\left(\frac{A_0}{t}\right)$
d) $A = 2ln\left(\frac{A_0}{10t}\right)$

Answer: A) Half-life is the time at which the remaining material, A, is half the amount of the original material, A_0. In others words when $t = t_{1/2}$, $A = \frac{A_0}{2}$. So, every multiple of $t = t_{1/2}$ decreases the amount of material by a factor of $\frac{1}{2}$. So, $A = A_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = A_0(2)^{-t/t_{1/2}}$. Since $t_{1/2} = 10$, $A = A_0(2)^{-t/10}$. For this reason, the answer is (a).